

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

(i) The slope of the bisector of the 1st & the 3rd quadrant is

- a) 0
- b) -1
- c) 1
- d) ∞

(ii) The distance of point (2,3) from x-axis is:

- a) 5
- b) 3
- c) 2
- d) 1

(iii) The length of the tangent from the point (1,2) to the circle $x^2 + y^2 - 2 = 0$ is:

- a) $\sqrt{2}$
- b) 1
- c) $\sqrt{6}$
- d) $\sqrt{3}$

(iv) Two lines, represented by $ax^2 + 2hxy + by^2 = 0$, where a, h, b are not all zero, will be orthogonal, if:

- a) $a - b = 0$
- b) $a + b = 0$
- c) $h = 0$
- d) $a = 0$

(v) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

- a) $\sqrt{g^2 + f^2 - c}$
- b) $\sqrt{g^2 + f^2 + c}$
- c) $\sqrt{g^2 - f^2 - c}$
- d) $\sqrt{g^2 + f^2 - c^2}$

(vi) $\frac{d}{dx} \ln x^2 =$:

- a) ax^{0-1}
- b) a/x
- c) $x^2 \ln x$
- d) $a^2 \ln a$

(vii) The necessary condition for f(x) to have extreme value is:

- a) $f'(x) = 0$
- b) $f''(x) = 0$

- c) $f(x) = 0$
- d) $f'(x) = 1$

(viii) $\int x^{-1} dx =$:

- a) $Xo + c$
- b) $1/x + c$
- c) $1/x^1 + c$
- d) $\ln x + c$

(ix) $\int \operatorname{cosec}^2 x dx =$:

- a) $-\operatorname{cosec} x + c$
- b) $-\cot x + c$
- c) $\operatorname{Cosec} x \cot x + c$
- d) $\ln \cot x + c$

(x) The distance between foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

- a) $\frac{2a}{e}$
- b) $2a$
- c) $2ae$
- d) $\frac{2b^2}{a}$

(xi) If $e = 3/2$, then the conic is a / an :

- a) Circle
- b) Ellipse
- c) Parabola
- d) hyperbola

MATHEMATICS

2019

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

(i) The centroid of triangle, whose two vertices are (2, 4) and (3, -4), is found to be (3, 1). Find its third vertex.

OR In what ratio is the line segment joining (1, 3) and (2, 7) divided by $3x + y = 9$?

(ii) Find the equation of the line which is perpendicular to $2x + 3y + 4 = 0$ and passes through (2, -1).

(iii) A line, whose y - Intercept is 1 less than its x - intercept form with the coordinate axis a triangle of area 6 square units. What is its equation?

(iv) A particle acted upon by the forces $4\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + \mathbf{j} - \mathbf{k}$, is displaced from the point (1, 2, 3) to point (5, 4, 1). Find the work done.

OR Calculate (a, b) for vectors $a = 2i + 3j + 4k$ and $b = i - j + k$

(v) Prove that: $[\bar{a} + \bar{b} \quad \bar{b} + \bar{c} \quad \bar{c} + \bar{a}] = 2[\bar{a} \quad \bar{b} \quad \bar{c}]$

ANALYTIC GEOMETRY (CONIC SECTIONS)

3.

(i) Find the equation of the circle of radius α which passes through the two points on the x - axis which are at a distance b from the origin.

(ii) Find the equation of the parabola with focus (3, 4) and directrix $x + y - 1 = 0$.

(iii) Find the equation of the ellipse whose center is at (0, 0); $e = 2/3$ latus rectum of length $20/3$ and major axis is along y - axis.

(iv) Find the eccentricity, foci and directrices of hyperbola $9x^3 - y^2 + 1 = 0$.

(v) The length of the major axis of an ellipse is 25 units and its foci are the points $(\pm 5, 0)$. Find its equation.

OR Show that the eccentricities e_1 and e_2 of two conjugate hyperbolae satisfy the relation: $e_1^2 + e_2^2 = e_1^2 e_2^2$

CALCULUS

4.

(i) Find the derivative, by the first principle at any point $x = \alpha$ in the domain $D(f)$ of the function

$$f(x) = \cos^2 x \quad \text{OR} \quad f(x) = x^{2/3}$$

(ii) Evaluate any two of the following:

(a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(b) $\lim_{x \rightarrow \infty} \frac{x^2 - 5x + 2}{5x^2 + 6x - 4}$

(c) $\lim_{x \rightarrow 0} \frac{3e^x - e^{-x} - 2}{x}$

(iii) Find the approximate value of $\sin 46^\circ$ using differentials.

(iv) To polynomial functions f and g are; defined by $f(x) = x^2 - 3x + 4$ and $g(x) = x + 1$, $\forall x \in \mathbb{R}$. Find fog. of an show that $f \circ g \neq g \circ f$.

OR Find the limit of the sequence $\frac{3}{2}, \frac{2}{3}, \frac{5}{4}, \frac{4}{5}, \dots$

(v) solve the differential equation: $2 + 2y \frac{dy}{dx} = 1 + 3x^2$, $y(2) = 1$

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

Note: answer any two questions from this section:

5. Evaluate any four of the following:

(i) $\int (x^3 + 1)^{7/5} x^5 dx$

(ii) $\int e^x \frac{1 + \sin x}{1 + \cos x} dx$

(iii) $\int_{-6}^{-3} \frac{\sqrt{x^2 - 9}}{x} dx$

(iv) $\int \frac{dx}{x^2 - x + 1}$

(v) $\int \frac{7x - 25}{(x - 3)(x - 4)} dx$

$$(vi) \int \frac{\sec x \tan x}{a+b \sec x} dx$$

6. (a) Find the centroid of the triangle the equation of whose sides are $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$.
(b) The Coordinates of two points A and B are (3, 4) and (5, 2) respectively. Find the coordinates of any point P if $PA = PB$ and the area of triangle PAB is 10 square units.
7. (a) Three vertices A, B and C of a triangle are (2, 1), (5, 2) and (3, 4) respectively. Find the coordinates of the circumcenter and the radius of the circumcircle of triangle ABC.
(b) Evaluate $\frac{dy}{dx}$ of any two of the following:
(i) $\sqrt{x^2 + y^2} = \ln(x^2 - y^2)$
(ii) $y = \ln \left[\frac{1-x^2}{1+x^2} \right]$
(iii) $X = a \cos^2 3\theta, y = b \sin^2 3\theta$

OR find the relative maximum and minimum values of the function $f(x) = e^x \sin x$

MATHEMATICS

2018

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

(i) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = :$

- a) e^x
- b) e
- c) 1
- d) 0

(ii) $\lim_{x \rightarrow 0} \frac{\sin cx}{x}:$

- a) 1
- b) $1/c$
- c) 0
- d) c

(iii) The least upper bound (l.u.b) of $\left\{-10, -5, 8, -\frac{1}{3}, 15, 21\right\}$ is:

- a) -10
- b) 8
- c) 15
- d) 21

(iv) The coordinate of centroid of the triangle whose vertices are (2, 8) (8, 2) and (9, 2) are:

- a) (3, 4)
- b) (19, 19)
- c) $\left[\frac{19}{3}, \frac{19}{3}\right]$
- d) $\left[\frac{1}{3}, \frac{1}{3}\right]$

(v) The inclination of x- axis is:

- a) 90°
 b) 45°
 c) 0°
 d) 270°
- (vi) The distance of point (3, 2) from x-axis is:
 a) $\sqrt{3}$ units
 b) 5 units
 c) 3 units
 d) 2 units
- (vii) If two or more straight line meet at one point, then the lines are said to be:
 a) Concurrent
 b) Parallel
 c) Perpendicular
 d) Coincident
- (viii) Some of the slopes of the pair of line $ax^2 + 2hxy + by^2 = 0$ is:
 a) $\frac{a}{b}$
 b) $\frac{h}{b}$
 c) $\frac{-h}{2a}$
 d) $\frac{-2h}{b}$
- (ix) $\frac{d}{dx} (\operatorname{cosec}^{-1} y)$:
 a) $\frac{-1}{y\sqrt{y^2-1}}$
 b) $\frac{1}{y\sqrt{1-y^2}}$
 c) $\frac{-1}{y\sqrt{1-y^2}}$
 d) $\frac{1}{y\sqrt{y^2-1}}$
- (x) An antiderivative of a function is also called:
 a) Definite integral
 b) Indefinite integral
 c) Summation
 d) Differential

MATHEMATICS

2018

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) Find the ratio in which y-axis divides the join of (-5, 3) and (8, 6). Also find the coordination of the point of Division.

- (ii) Reduce the equation $2x - 3y + 4 = 0$ into:
 (a) Perpendicular form
 (b) Slope-intercept form
- (iii) Find the coordinates of the foot of the perpendicular from $(-2, 5)$ to $3x + y + 11 = 0$.
- (iv) Calculate $\sin(\vec{a}, \vec{b})$ for vector $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.
- OR Find $\cos\theta$, when θ is the single between the vectors $2\hat{i} + \hat{j} + 3\hat{k}$ and $-\hat{i} + 4\hat{j} - 2\hat{k}$.
- (v) Simplify: $[\vec{a}, 2\vec{b}, -3\vec{c}, -2\vec{a} + \vec{b} + \vec{c}]$

(Analytic Geometry)(Conic Sections)

- 3.**
- (i) Find the equation of the circle concentric with the circle $x^2 + y^2 + 6x - 10y + 33 = 0$ and touching the line $y = 0$
- (ii) Prove that the product of abscissas, of the point where the straight line $y = mx$ meets the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is equal to $\frac{c}{1 + m^2}$.
- (iii) Prove that the focal radius of the point (a, b) on the parabola $x^2 = 4ay$ equals $|a + b|$.
 OR Find the equation of the parabola whose vertex is $(3, 2)$ and the ends of focal chord are $(5, 6)$ and $(5, -2)$.
- (iv) Find the eccentricity of the ellipse whose axes are 32 and 24.
- (v) Find the equation of the rectangular hyperbola with center at $(0, 0)$ and vertices (0 ± 3) .
 OR Show that the eccentricities. e_1 the e_2 conjugate hyperbolas satisfy the relation $e_1^2 + e_2^2 = e_1^2 e_2^2$

CALCULUS

- 4.**
- (i) Find the derivative, by the first principle, at $x = a$, in the domain $D(f)$, of the function $f(x) = \sec x$
 OR $f(x) = 3x^2 + x$.
- (ii) Evaluate any two of the following:
 a) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$
 b) $\lim_{x \rightarrow 0} \frac{e^{mx} - e^{nx}}{x^2}$
 c) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$
- (iii) Calculate $\log_{10}(10.1)$ using differentials, given that $\log_{10} e = 0.4343$.
- (iv) Find the area above the x-axis under the curve $y = 2e^{3x}$, between ordinates $a = 2$ And $b = 5$.
- OR Solve the differential equation: $\frac{dy}{dx} = \sqrt{x}, y, x = 100, y = 9$
- (v) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = \begin{cases} 0, & \text{when } x \in \mathbb{Q} \\ 1, & \text{when } x \in \mathbb{R} - \mathbb{Q} \end{cases}$, \mathbb{Q} being the set of rationals
 Find $f(\sqrt{3}), f(\pi), f(\frac{1}{2}), f(2)$ and the value of f at 1.5.

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

Note: answer any two questions from this section:

- 5.** Evaluate any four of the following:
 i. $\int_0^2 (x^2 + bx + c) - 2/3 \left(x + \frac{b}{2}\right) dx$

- ii. $\int \tan^{-1} x \, dx$
- iii. $\int \frac{dx}{\sqrt{5+4x-x^2}}$
- iv. $\int \frac{7x-25}{(x-3)(x-4)} \, dx$
- v. $\int_0^2 \frac{y^3 \, dy}{\sqrt{16-y^2}}$
- vi. $\int \frac{\sec x \tan x}{a+b \sec x} \, dx$
6. (a) The coordinates of two points A and B are (3, 4) and (5, -2) respectively. Find the coordinates of any point P if PA = PB and the area of triangle PAB is 10 square units-
OR Find the combined equation of the pair of lines through the origin which are perpendicular to the line represented by $3x^2 + 7xy + 2y^2 = 0$.
- (b) Find the equation of the tangents at the end of the latus rectum of the parabola $y^2 = 4ax$
7. (a) Find the equation of the straight line which passes through the point (-3, 2) and is such that the portion of it between axes is divided by the point in the ratio 1: 2.
- (c) Evaluate $\frac{dy}{dx}$ of any two of the following:
- (i) $y = \sqrt{a^2 - x^2} + \sin \sqrt{1 + x^2}$
- (ii) $x^y \cdot y^x = 10$
- (iii) $x = \int t \, dt + \sin^{-1} t, y = e + \cos t$
- OR Find a right angled triangle of maximum area with a hypotenuse of length h.

MATHEMATICS

2017

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:
- (i) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:
- a) $\sqrt{g^2 + f^2 + c}$
- b) $\sqrt{c - g^2 - f^2}$
- c) $\sqrt{g^2 - f^2 - c}$
- d) $\sqrt{g + f - c}$
- (ii) The length of latus rectum of the parabola $x^2 = -28y$ is:
- (a) 7
- (b) 28
- (c) 192
- (d) -7
- (iii) The equations of the directrices of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are:
- a) $x = \pm a$
- b) $y = \pm a$
- c) $x = \pm a/e$

- d) $y = \pm 1/a$
- (iv) The vertex of the parabola $(x + 2)^2 = 4(y - 2)$ is:
- $(-2, -2)$
 - $(3, -2)$
 - $(-2, 3)$
 - $(-2, 2)$
- (v) If $b^2 = a^2 (e^2 - 1)$, then the conic is:
- Parabola
 - Ellipse
 - Hyperbola
 - Circle
- (vi) $\vec{a} \cdot \vec{b} \times \vec{c} =$:
- $\vec{a}\vec{b}\vec{c}$
 - $\vec{a}x\vec{b}x\vec{c}$
 - $\vec{a} \cdot \vec{b} \cdot \vec{c}$
 - $\vec{a} \times \vec{b} \cdot \vec{c}$
- (vii) If \vec{a} and \vec{b} are perpendicular then $\vec{a} \cdot \vec{b} =$:
- 1
 - 1
 - 0
 - $\pi/2$
- (viii) if $f(x) = \sin x \cos x$, then $f(x)$ is:
- even
 - odd
 - both even and odd
 - neither even nor odd
- (ix) if $f : \mathbb{R} \rightarrow \mathbb{R}$ is given $f(x) = \sqrt{x}$, then $f(16) =$:
- 4
 - 6
 - 4
 - 8
- (x) Every linear equation represents a:
- Straight line
 - Circle
 - Curve
 - Point

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) Prove that the points, whose coordinates are (5, 1), (1, -1) and (11, 4), lie on a straight line. Find the Intercepts made by this line on the axes.

OR Find the measure of the angle from a line with slope $-2/3$ to:

- a) y - axis
b) x -axis.
- (ii) Find the value of k when the vertices of the triangle are the points (2, 9), (-2, 1) & (k, 3) & its area is 28 square unit
- (iii) The gradient of one of the lines $ax^2 + 2hxy + by^2 = 0$ is five times that of the other. Show that $5h^2 = 9ab$.
- (iv) Find the unit vector perpendicular to both of the vectors $\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 4\hat{k}$. Also calculate sine of the angle between the given vectors.
OR Find $\cos\theta$, where θ is the angle between vectors $2\hat{i} - \hat{j} + 3\hat{k}$ and $-\hat{i} - 4\hat{j} + 2\hat{k}$.
- (v) Resolve the vector $\vec{i} = (6, 8, -6)$ in the direction of vectors $p_1 = (1, -1, 2)$, $p_2 = (2, 2, -1)$ and $p_3 = (3, 7, -7)$

ANALYTIC GEOMETRY (CONIC SECTIONS)

3.

- (i) Find the equation of the circle containing the points (-1, -1) and (3, 1) and with the center on the line $x - y + 10 = 0$.
- (ii) Show that lines $x = 5$ and $y = 7$ both touch the circle $x^2 + y^2 - 4x - 8y + 11 = 0$.
- (iii) Find the equation of the parabola whose focus is (3, 4) and the directrix is the line $x + y - 1 = 0$.
- (iv) Find the center foci and equations of directories of the hyperbola $\frac{(x+2)^2}{4} - \frac{(y-1)^2}{9} = 1$
- (v) Find the eccentricity of an ellipse whose latus rectum is equal to half of its major axes.
OR The length of the major axis of an ellipse is 25 & its foci are the points $(\pm 5, 0)$. Find the equation of the ellipse.

CALCULUS

4.

- (i) Find the derivative, by the first principle, at $x = a$ of the function $f(x) = \tan x$ OR $f(x) = \sin 2x$.
- (ii) Evaluate any Two of the following:
- a) $\lim_{x \rightarrow 0} \frac{\sin^2(\frac{x}{3})}{x^2}$
- b) $\lim_{x \rightarrow 0} \frac{3e^x - e^{-x} - 2}{x}$
- c) $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$ OR $\lim_{x \rightarrow 0} \left(1 + \frac{3}{t}\right)$
- (iii) Calculate the approximate value of $\cos 46^\circ$ OR $\cos 59^\circ$ using differentials.

- (iv) Two polynomial functions f and g are defined by $f(x) = x^2 - 3x + 4$ and $g(x) = x + 1, \forall x \in \mathbb{R}$. Show that $f \circ g$ is not equal to $g \circ f$.
- (v) Find the area above x -axis, under the curve, $n \frac{x^2}{4} + \frac{y^2}{9} = 1$ between the ordinates $x = -1$ and $x = 1$.

OR Solve the differential equation: $\frac{dy}{dx} = \frac{(1+x^2)}{\sin 3y}$

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

5. Evaluate any four of the following:

(i) $\int \frac{3x^2 + 1}{(x^3 + x + 6)^{\frac{1}{2}}} dx$ OR $\int_{-2}^{-1} x\sqrt{2x^2 + 3} dx$ (ii) $\int \frac{x+3}{x^2 + 2x + 5} dx$

(iii) $\int \sqrt{4-x^2} dx$

(iv) $\int \frac{2x-1}{x(x-1)(x-3)} dx$

(Using trigonometric substitution)

(v) $\int x^2 \tan^{-1} x dx$

(vi) $\int_0^{\pi/2} \cos^4 x dx$

6. (a) The area of a triangle is 8 square units, two of its vertices are the points A (1,-2) and B (2, 3) and the third vertex C lies on the line $2x + y - 2 = 0$. Find the coordinates of the vertex C.

(b) Prove that the line $tx + my + n = 0$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ have just one point in common, if $a^2 t^2 + b^2 m^2 - n^2 = 0$

7. (a) A is two-thirds the way from (1, 10) to (-8, 4) and B is the mid-point of (0, -7) and (6, -11). Find the distance AB.

(b) Evaluate $\frac{dy}{dx}$ of any two of the following:

- i. $\sqrt{x^2 + y^2} = \ln(x^2 - y^2)$
- ii. $y = \cos^{-1} \left(\frac{2x}{1-x^2} \right)$
- iii. $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$

OR Find the maximum and minimum value of the function $f(x) = x^3 - 9x^2 + 15x + 3, \forall x \in \mathbb{R}$.

MATHEMATICS

2016

Time: 20 Minutes

Max. Marks: 20

SECTION "A" (MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

- (i) If $f: [-1, 5] \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$ then $f(-2) =$
- (a) 4
 - (b) -2
 - (c) Undefined

- (d) -4
- (ii) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} =$:
- (a) $e^z \ln a$
 (b) 1
 (c) $\ln x$
 (d) $\frac{1}{\ln x}$
- (iii) The slope of a straight line which bisect the first and third quadrants is:
- (a) 1
 (b) 0
 (c) -1
 (d) ∞
- (iv) The area of triangle whose vertices are (0,0) , (2,0) and (4, 0) is:
- (a) 8 sq. units
 (b) 4 sq. units
 (c) 2 sq. units
 (d) 1 sq. units
- (v) If the equation of a straight line is $3x - y + 5 = 0$, then the point (1, 2) lies:
- (a) above the line
 (b) below the line
 (c) on the line
 (d) on both sides of the line
- (vi) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:
- a) $\sqrt{g^2 + f^2 + c}$
 b) $\sqrt{c - g^2 - f^2}$
 c) $\sqrt{g^2 - f^2 - c}$
 d) $\sqrt{g + f - c}$
- (vii) The length of latus rectum of the parabola $x^2 = -28y$ is:
- a) 7
 b) 28
 c) 192
 d) -7
- (viii) The equations of the directories of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are:
- a) $x = \pm a$
 b) $y = \pm a$
 c) $x = \pm a/e$
 d) $y = \pm 1/a$
- (ix) The vertex .of the parabola $(x + 2)^2 = 4(y - 2)$ is:
- a) (-2, -2)
 b) (3, -2)
 c) (-2, 3)
 d) (-2, 2)
- (x) The point of concurrency of the medians of a triangle is called:
- (a) In Centre
 (b) Centroid
 (c) Orthocenter

MATHEMATICS

2016

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) Find the ratio in which y-axis divides the join of (-5, 3) and (8, 6). Also find the coordinates of the point of division.
- (ii) Find the equation of the line which passes through the point (-2, -4) and has sum of Intercepts equal to 3
- (iii) Find the value of k for which the two lines $(k,1)x + ky - 5 = 0$, $kx + (2k - 1)y + 7 = 0$ Intersect at a point lying on the axis of x.
- (iv) Prove that the point (5, -7.5) lies outside the circle whose equation is $x^2 + y^2 - 4x + 2y = 44$
- (v) Find the equation of a circle with center at the point (1, -1) and touching the straight line $5x + 12 = 7$.
- (vi) Find the equation of the parabola with focus (2, 3) and directrix $y - 5 = 0$.
- (vii) The length of the major axis of an ellipse is 25 units and its foci are the points $(\pm 5, 0)$. Find its equation.
- (viii) Find the eccentricity foci, directories and length of the latus rectum of the hyperbola $9x^2 - y^2 + 1 = 0$.
- (ix) A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$ and its displaced from the point $\hat{i} + 2\hat{j} - 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} - 3\hat{k}$ find the work done by forces.
- (x) Prove that $[\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{c} + \vec{a}] = 2 [\vec{a} \vec{b} \vec{c}]$

CALCULUS

Note: Attempt 3 part questions from this section.

3.

- (i) Find the limit of the sequence $\frac{1.2}{3.4}, \frac{3.4}{5.6}, \frac{5.6}{7.8}, \dots$
- (ii) Find the derivative by the first principles at $x = a$ in the domain $D(f)$ of the function $f(x) = \tan x$
OR Evaluate any two of the following

$$(a) \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x}$$

$$(b) \quad \lim_{x \rightarrow 0} \frac{3e^x - e^{-x} - 2}{x}$$

$$(c) \quad \lim_{x \rightarrow 1} \left[\frac{1}{1-x} - \frac{3}{1-x^3} \right]$$

- (iii) Find $\frac{dy}{dx}$ of any two of the following:
 - a) $y = \tan^2 x + \sec x$
 - b) $2x^2 - 3xy + y^2 = 5$
 - c) $x = \ln t + \cos t, y = e^t + \sin t$
- (iv) Using differentials, find the approximate value of $\cos 44^\circ$
- (xv) Evaluate any two of the following:

$$(a) \int \frac{dx}{\sqrt{1+x} - \sqrt{x}} \quad (b) \int \frac{\cos \ln x \, dx}{x(3 - \sin \ln x)^{1/2}} \quad (c) \int \ln x \, dx$$

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

4.

- (a) Find the equation of the locus of a moving point such that the slope of line joining the point to A (1, 3) is three times the slope of the line joining the point to B (3, 1).
 (b) The point (2, -5) is a vertex of a square, one of whose sides lies on the line $x - 2y - 7 = 0$. Calculate the area of the square.

5.

- (a) if $y = ae^x + be^{2x} + ce^{3x}$, show that $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$.
 (b) Find the maximum and minimum values of the function $f(x) = e^x \sin x$.

6. (a) Evaluate any two of them:

$$(i) \int \frac{\tan x}{\ln(\cos x)} \, dx \quad (ii) \int \frac{2x}{(1-x^2)(3+x^2)} \, dx$$

$$(iii) \int_1^2 (x+1)\sqrt{x^2+2x+2} \, dx$$

OR (i) Find the area above the x-axis under the curve $y = \tan x$ between the ordinates $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$.

(ii) Solve the differential equation $\frac{dy}{dx} = \sqrt{xy}$ where $y(9) = 100$.

(b). Prove that parabolas $x^2 = 4ay$ and $y^2 = 4bx$ intersect at angle $\theta = \tan^{-1} \frac{3}{2} \left[\frac{a^{1/3} b^{1/3}}{a^{2/3} + b^{2/3}} \right]$

MATHEMATICS

2015

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

- (i) If $b^2 = a^2(1 - e^2)$, then the conic is:
 a) Circle
 b) Parabola
 c) Hyperbola
 d) Ellipse
- (ii) In the parabola $y^2 = 4ax$, $|4a|$ represents:
 a) Focus
 b) Vertex
 c) Axis
 d) length of latus rectum

- (iii) If vectors \vec{a} and \vec{b} are perpendicular, then $\vec{a} \cdot \vec{b} =$:
- 1
 - 1
 - 0
 - $\pi/2$
- (iv) Magnitude of a vector $(1, -\sqrt{3}, -\sqrt{5})$ is:
- 9
 - 3
 - $\sqrt{3}$
 - $\sqrt{5}$
- (v) The function $f(x) = \cos x$ is:
- Even
 - Odd
 - Modulus
 - Inverse
- (vi) If $f: [-1, 5] \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$ then $f(-2) =$:
- 4
 - 2
 - Undefined
 - 4
- (vii) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} =$:
- $e^2 \ln a$
 - 1
 - $\ln x$
 - $\frac{1}{\ln x}$
- (viii) The slope of a straight line which bisect the first and third quadrants is:
- 1
 - 0
 - 1
 - ∞
- (ix) The area of triangle whose vertices are $(0,0)$, $(2,0)$ and $(4, 0)$ is:
- 8 sq. units
 - 4 sq. units
 - 2 sq. units
 - 1 sq. units
- (x) If the equation of a straight line is $3x - y + 5 = 0$, then the point $(1, 2)$ lies:
- above the line
 - below the line
 - on the line
 - on both sides of the line

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 7 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) Find the co-ordinates of the foot of perpendicular from $(-2, 5)$ to $3x + y + 11 = 0$.
- (ii) Using slopes, prove that $(12, 8)$, $(-2, 6)$ and $(6, 0)$ are the vertices of a right triangle.
- (iii) Show that the points $(5, 1)$, $(1, -1)$ and $(11, 4)$ lie on a straight line. Find its equation.
- (iv) The area of a triangle is 8 square units, two of its vertices are the points $A(1, -2)$ and $B(2, 3)$ and the third vertex C lies on the line $2x + y - 2 = 0$. Find co-ordinates of vertex C .
- (v) Find the equation of the circle containing the points $(-1, -1)$ and $(3, 1)$ and the line $x - y + 10 = 0$ passing through the center of the circle.
- (vi) Find the eccentricity of an ellipse whose length of latus rectum is half of its major axis.
- (vii) Prove that the two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other if $\frac{1}{f^2} + \frac{1}{g^2} = \frac{1}{c}$.
- (viii) Find center, foci and equation of directrices of hyperbola $\frac{(x-1)^2}{4} - \frac{(y-2)^2}{9} = 1$.

OR Find the equation of circle whose diameter is the length of latus rectum of parabola $x^2 = 36y$.

- (ix) Find constant 'a' such that the sets of vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$, $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar.

OR Calculate $\sin \vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

- (x) Resolve the vector 'a' in a plane in the direction of $P_1 P_2$, where $a = (9, 4)$, $P_1 = (2, -3)$, $P_2 = (1, 2)$.

CALCULUS

Note: Attempt 3 part questions from this section.

3.

- (i) Find the derivative by First Principia at $x = a$ in the domain of $D(f)$ of $f(x) = \operatorname{cosec} x$ OR $f(x) = 3x^2 + x$.
- (ii) Evaluate any two of the following:

$$(a) \lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin x} \quad (b) \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 16} - 4}{x} \quad (c) \lim_{x \rightarrow \infty} \frac{2t^2 - 2t + 1}{t^2 + 4}$$

OR Determine the limit of the sequence $\frac{1}{1.2}, \frac{1}{2.3}, \frac{1}{3.4}, \dots$

- (iii) Calculate the approximate value of $\log_{10}(10.1)$, given that $\log_{10} e = 0.4343$.
- (iv) Calculate the approximate value of $\sin 46^\circ$ by using differential.
- (v) Evaluate any two of the following:

a) $\int (ax^2 + bx + c)^{-2/3} \cdot \left[x + \frac{b}{c} \right] dx$

b) $\int \cos 5x \cdot \sin 3x dx$

c) $\int \frac{x^2}{\sqrt{1-x^6}} dx$

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

- (a) Find the equation of the two straight lines passing through (3, -2) and inclined at 60° to the line $\sqrt{3}x + y = 1$,
- (b) Find the equation of circle passing through the focus of parabola $x^2 + 8y = 0$ and foci of ellipse, $16x^2 + 25y^2 = 400$.

OR Prove that the line $lx + my + n = 0$ and the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ have just one point in common if $a^2l^2 + b^2m^2 - n^2 = 0$.

4.

- (a) Evaluate any two of the following:

- i. $\int \frac{7x - 25}{(x-3)(x-4)} dx$
- ii. $\int x \cdot \tan^{-1}x dx$
- iii. $\int \frac{x^3}{\sqrt{a^2 - x^2}} dx$
- iv. $\int_0^2 \frac{dx}{9 - x^2}$

- (b) If the line $y - 2 = 0$, intersects the pair of lines $x^2 - 7xy + 2y^2 = 0$ in A and B and 'O' be the origin. Find the area of triangle OAB.

5.

- (a) (i). Find the area under the curve $y = 3 \sin x$ between the ordinates $x = 0$ and $x = \pi/3$
(ii) Solve the differential equation: $\frac{dy}{dx} = x \cos^2 y$.
- (b) Find the extreme value of the function f given by $f(x) = x(x-1)(x-2)$, $\forall x \in \mathbb{R}$.

MATHEMATICS

2014

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1.

- (i) Magnitude of the vector $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ is:

- a) 13
- b) $\sqrt{12}$
- c) $\sqrt{14}$
- d) $\sqrt{11}$

- (ii) $\lim_{x \rightarrow 0} \frac{\sin 2x}{x}$

- a) 0
- b) 1
- c) $\frac{1}{2}$
- d) 2

- (iii) A function $f(x) = |x| - x^2$ is:

- a) Odd
- b) Even
- c) Neither even nor odd

- d) Modulus
- (iv) The vertex of the parabola $(x + 2)^2 = 4(y - 2)$ is:
- (-2, -2)
 - (3, -2)
 - (-2, 3)
 - (-2, 2)
- (v) If $b^2 = a^2 (e^2 - 1)$, then the conic is:
- Parabola
 - Ellipse
 - Hyperbola
 - Circle
- (vi) $\vec{a} \cdot \vec{b} \times \vec{c} =$:
- $\vec{a}\vec{b}\vec{c}$
 - $\vec{a}x\vec{b}x\vec{c}$
 - $\vec{a} \cdot \vec{b} \cdot \vec{c}$
 - $\vec{a} \times \vec{b} \cdot \vec{c}$
- (vii) If \vec{a} and \vec{b} are perpendicular then $\vec{a} \cdot \vec{b} =$:
- 1
 - 1
 - 0
 - $\pi/2$
- (viii) if $f(x) = \sin x \cos x$, then $f(x)$ is:
- even
 - odd
 - both even and odd
 - neither even nor odd
- (ix) if $f : \mathbb{R} \rightarrow \mathbb{R}$ is given $f(x) = \sqrt{x}$, then $f(16) =$:
- 4
 - 6
 - 4
 - 8
- (x) Every linear equation represents a:
- Straight line
 - Circle
 - Curve
 - Point

MATHEMATICS

2014

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 7 questions from this section. At least three questions from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) If the line through (2, 5) and (-3, -2) is perpendicular to the through (4, -1) and (x, 3), find x.
- (ii) Find the combined equation of the pair of lines through the origin which are perpendicular to the lines Represented by $6x^2 - 13xy + 6y^2 = 0$.
OR The gradient of one of the lines $ax^2 + 2hxy + by^2 = 0$ is two times that of the other. Show that $8h^2 = 9ab$
- (iii) Find the distance two parallel lines $5x - 12y + 10 = 0$ and $5x - 12y - 16 = 0$.
- (iv) Find the equation of the parabola having focus (-5, 3) and directrix $y - 7 = 0$.
- (v) Find the equation of the circle which is concentric with the circle $x^2 + y^2 - 8x + 12y - 12 = 0$ and passes through the point (5, 4).
- (vi) Find the center, focus and eccentricity of the ellipse $\frac{(x-3)^2}{25} + \frac{(y+1)^2}{9} = 1$.
- (vii) Find the equation of the hyperbola with focus (8, 0) and directrix $x = 4$.
- (viii) A particle, acted upon by the forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} - \hat{j} - \hat{k}$ is displaced from the point (1, 2, 3) to point (5, 4, 1). Find the work done.
- (ix) Find the unit vector perpendicular to both the vectors $\hat{i} + 2\hat{j} + 2\hat{k}$ and $3\hat{i} - 2\hat{j} - 4\hat{k}$. Also find sine of the angle them.
OR Simplify: $[\vec{a}, 2\vec{b} - 3\vec{c}, -2\vec{a} + \vec{b} + \vec{c}]$
- (x) Find the equation of the line passing through the intersection of the lines $2x + 3y + 1 = 0$, $3x - 4y - 5 = 0$ and passing through the point (2, 1).
OR Find the equation of the locus of the points which are equidistance from the point (0, 3) and the line $y + 3 = 0$.

CALCULUS

Note: Attempt any three part question from this section.

3.

- (i) Find the derivative by first principle at $x = a$ is the domain of D(f) of $f(x) = \cot x$ OR $f(x) = 3x^3 - x$.
- (ii) Evaluate any two of the following:
- a) $\lim_{x \rightarrow a} \frac{x^m - a^m}{x^n - a^n}$
- b) $\lim_{x \rightarrow a} \frac{\sqrt{x^2 + 16} - 4}{x}$
- c) $\lim_{x \rightarrow a} \frac{\tan x - \sin x}{x}$
- OR** Find the limit of the sequence $\frac{1.2}{3.4} + \frac{3.4}{5.6} + \frac{5.6}{7.8} + \dots$
- (iii) Find $\frac{dy}{dx}$ of any two of the following:
- a) $e^x \ln y = \sin^{-1} y$
- b) $\sqrt{x^2 + y^2} = \ln(x^2 - y^2)$
- c) $x^y \cdot y^x = 10$
- (iv) Evaluate any two of them:

(a) $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx$ (b) $\int x \ln x dx$

(c) $\int \frac{\sin(\ln x) dx}{x(3 - \cos \ln x)^{\frac{1}{2}}}$

- (v) Using differential, show that $\sqrt{x} + \frac{1}{2\sqrt{x}} dx$. Hence, find the value of $\sqrt{3.9}$
 OR Calculate an approximate value of $\tan 44^\circ$ by using differential.

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

4.

- (a) Find the equation of circle passing through the focus of parabola $x^2 + 8y = 0$ and foci of ellipse $16x^2 + 25y^2 = 400$.

OR Determine the vertex, focus and the equation of directrix of $y^2 + 4y + 3x - 92 = 0$.

- (b) Find the condition that conics $ax^2 + by^2 = 1$ and $a! x^2 + b! y^2 = 1$ cut each other orthogonally.

5.

- (a) Evaluate any Two of the following:

$$(i) \int_0^1 \frac{dx}{\sqrt{4-x^2}} \quad (ii) \int \frac{2x dx}{\cos^2 2x} \quad (iii) \int \frac{\sin x dx}{(1+\cos x)(2+\cos x)}$$

- (b) Show that the eccentricities e_1 and of two conjugate hyperbola satisfy the relation $e_1^2 + e_2^2 = e_1^2 e_2^2$.

OR If $y = \sqrt{5}x + k$ is tangent to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. What is k ?

6.

- (a) (i) Solve the differential equation: $2+2y\frac{dy}{dx} = 1 + 3x^2$, $y(2) = 1$

OR $\frac{du}{dv} = \sqrt{u \cdot v}$ $u = 100$, $v = 9$

- (ii) Find the area under curve $y = x - \frac{5}{x^2}$ between the ordinates $x = 2$, $x = 4$.

- (b) Find the relative maximum and minimum value of the function $\sin x$ OR $f(x) = x^3 - 9x^2 + 15x + 3$.

MATHEMATICS

2013

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1.

- (i) The slope of the bisector of the 1st & the 3rd qiatramt os"

- a) 0
- b) -1
- c) 1
- d) ∞

- (ii) The distance of point (2,3) from x - axis is:

- a) 5
- b) 3
- c) 2

d) 1

(iii) The length of the tangent from the point (1,2) to the circle $x^2 + y^2 - 2 = 0$ is:

- a) $\sqrt{2}$
- b) 1
- c) $\sqrt{6}$
- d) $\sqrt{3}$

(iv) Two lines, represented by $az^2 + 2hxy + by^2 = 0$, where a, h, b are not all zero, will be orthogonal, if:

- a) $a - b = 0$
- b) $a + b = 0$
- c) $h = 0$
- d) $a = 0$

(v) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

- a) $\sqrt{g^2 + f^2 - c}$
- b) $\sqrt{g^2 + f^2 + c}$
- c) $\sqrt{g^2 - f^2 - c}$
- d) $\sqrt{g^2 + f^2 - c^2}$

(vi) $\frac{d}{dx} \ln x^2 =$:

- a) ax^{0-1}
- b) a/x
- c) $x^2 \ln x$
- d) $a^2 \ln a$

(vii) The necessary condition for $f(x)$ to have extreme value is:

- a) $f'(x) = 0$
- b) $f''(x) = 0$
- c) $f(x) = 0$
- d) $f''(x) = 1$

(viii) $\int x^{-1} dx =$:

- a) $Xo + c$
- b) $1/x + c$
- c) $1/x^1 + c$
- d) $\ln x + c$

(ix) $\int \operatorname{cosec}^2 x dx =$:

- a) $-\operatorname{cosec} x + c$
- b) $-\cot x + c$
- c) $\operatorname{Cos} ecx \cot x + c$
- d) $\ln \cot x + c$

(x) The distance between foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

- a) $\frac{2a}{e}$
- b) $2a$
- c) $2ae$
- d) $\frac{2b^2}{a}$

(xi) If $e = 3/2$, then the conic is a / an :

- a) Circle
- b) Ellipse
- c) Parabola
- d) hyperbola

MATHEMATICS

2013

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 7 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) If the line through (2, 5) and (-3, -2) is perpendicular to the line through (4, -1) and (x, 3), find x.
- (ii) Find the equation of the line which passes through the point (-3,-4) and has the sum of intercepts equal to 1.
- (iii) Find the value of K when the vertices of the triangle are the points (2, 6), (6, 3) & (4, k) and area is 15 units.
- (iv) The gradient of one of the lines $ax^2 + 2hxy + by = 0$ is five times that of the other. Show that $5h^2 = 9ab$.
- (v) Find equation of the circle touching each of the axes in 4th quadrant at a distance of 5 units from the origin.
- (vi) Find equation of the circle which is concentric with the circle $x^2 + y^2 - 8x + 12y + 15 = 0$ and passes through the point (5, 4).
- (vii) Determine the vertex, focus and equation of directrix of the curve $x^2 + 4x + 4y - 12 = 0$.
- (viii) Find equation of the hyperbola having focus (8, 0) and directrix $x = 4$. OR Find the eccentricity, foci and equations of directories of $25x^2 + 9y^2 = 225$.
- (ix) Find $\sin(\vec{a}, \vec{b})$ where $\vec{a} = \hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = -3\hat{i} - 3\hat{k}$.
- (x) Find volume of the parallelepiped whose three adjacent edges are represented by the vectors $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{j} + 2\hat{k}$.

CALCULUS

Note: Attempt 3 part questions from this section.

3.

- (i) Find the derivative by the first principles at $x = a$ in the domain $D(f)$ of $f(x) = \sin 2x$.
- (ii) Evaluate any Two of the following:

$$(a) \lim_{\theta \rightarrow \frac{\pi}{2}} \frac{\cot \theta - \cos \theta}{\cos^3 \theta} \quad (b) \lim_{x \rightarrow 0} \frac{e^{mx} - e^{nx}}{x}, m, n \in \mathbb{R}$$

$$(b) \lim_{x \rightarrow 0} \frac{x}{\sqrt{1 - \cos x}} \quad \text{OR} \quad \lim_{x \rightarrow 1} \left[\frac{1}{1-x} - \frac{3}{1-x^3} \right]$$

(iii) Find $\frac{dy}{dx}$ of any two of the following:

(a) $y = e^{\sin x + \cos x}$

(b) $y = (\sin^{-1} x)^3$

(c) $y = \sqrt[5]{(x^2 + 2x + 3)}$

(iv) Evaluate any two of the following:

a) $\int \frac{dx}{\sqrt{1+x+\sqrt{x}}}$

b) $\int \frac{dx}{9-x^2}$

c) $\int \cos 4x \cos 2x dx$

(v) Using diff. show that $\sqrt{x + \Delta x}$ can be approximated to $\sqrt{x} + \frac{1}{2\sqrt{x}} \Delta x$. Hence find the value of $\sqrt{9.1}$.

OR Find the nth term and the limit of the sequence: $\frac{1.3}{2.4}, \frac{3.5}{4.6}, \frac{5.7}{6.8}, \dots$ where dot “.” Represents multiplication

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

4.

(a) Equation of a curve is given by $x^2 - 2xy + y^2 + 2x - 4 = 0$, find slope of the curve at the point (2, 2).

(b) Find equation of the circle containing the (-1, -1) and (3, 1) and with the center on the line $x - y + 10 = 0$

5.

(a) Evaluate any Two of the following:

(i) $\int \frac{\cos x dx}{\sin x(2 + \sin x)}$

(ii) $\int \frac{\sec x \tan x dx}{a + b \sec x}$

(iii) $\int_0^{\frac{\pi}{2}} \cos^4 x dx$

(b) Prove that line $lx + my + n = 0$ & the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ have just one point in common if $a^2 l^2 + b^2 m^2 - n^2 = 0$.

6.

(a) (i). Find the area under the curve $y = 3x^4 - 2x^3 + 1$, above x-axis and between $x = 1$ and $x = 2$.

(ii). Solve the differential equation $dy / dx = y^2 \sin x$.

(b) Find the relative maximum and relative minimum values of the function $f(x) = \frac{\ln x}{x}$.

SECTION "A"(MULTIPLE CHOICE QUESTION)

1.

- (i) If $b^2 = a^2 (e^1 - 1)$, then the conic is:
- Parabola
 - Ellipse
 - Hyperbola
 - Circle
- (ii) $\vec{a} \cdot \vec{b} \times \vec{c} =$:
- $\vec{a}\vec{b}\vec{c}$
 - $\vec{a}\vec{x}\vec{b}\vec{x}\vec{c}$
 - $\vec{a} \cdot \vec{b} \cdot \vec{c}$
 - $\vec{a} \times \vec{b} \cdot \vec{c}$
- (iii) If \vec{a} and \vec{b} are perpendicular then $\vec{a} \cdot \vec{b} =$:
- 1
 - 1
 - 0
 - $\pi/2$
- (iv) if $f(x) = \sin x \cos x$, then $f(x)$ is:
- even
 - odd
 - both even and odd
 - neither even nor odd
- (v) if $f : \mathbb{R} \rightarrow \mathbb{R}$ is given $f(x) = \sqrt{x}$, then $f(16) =$:
- 4
 - 6
 - 4
 - 8
- (vi) Every linear equation represent a:
- Straight line
 - Circle
 - Curve
 - Point
- (vii)
- (viii) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:
- $\sqrt{g^2 + f^2 + c}$
 - $\sqrt{c - g^2 - f^2}$
 - $\sqrt{g^2 - f^2 - c}$
 - $\sqrt{g + f - c}$
- (ix) The length of latus rectum of the parabola $x^2 = -28y$ is:
- 7
 - 28

c) 192

d) -7

(x) The equations of the directories of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are:

a) $x = \pm a$

b) $y = \pm a$

c) $x = \pm a/e$

d) $y = \pm 1/a$

MATHEMATICS

2012

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 7 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) A straight line passes through the points A (-12, -13) and B (-2, -5). Find the point on the line whose ordinate is -1
- (ii) Find the equation of a line which passes through the point (-1, 2) and has sum of equal to 2.
- (iii) Find the equation of a line through the intersection of the lines $7x - 13y + 46 = 0$, $19x + 11y - 41 = 0$ and passing through the point (3, 1) by using K-method.
- (iv) The point (-2, 1) is a vertex of a rectangle whose sides lie on the lines $3x - 2y - 5 = 0$, $2x + 3y + 7 = 0$. Find area of the rectangle.
- (v) Find the equation of circle concentric with the circle $x^2 + y^2 + 6x - 10y + 33 = 0$ and touching the y-axis.
- (vi) Prove that the straight line $y = x + c\sqrt{2}$ touches the circle $x^2 + c^2$, and find point of contact.
- (vii) Find the equation of parabola with focus (2, 3) and directrix $y - 5 = 0$.
- (viii) Find the equation of ellipse whose center is at (0, 0), $e = \frac{2}{3}$, latus rectum of length $\frac{20}{3}$ and major axis is along x-axis.
OR Find the eccentricity, foci and equations of directories of hyperbola $9x^2 - y^2 + 1 = 0$.
- (ix) Find the unit vector perpendicular to both the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$
- (x) A particle acted upon the forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point P (1, 2, 3) to the point Q (5, 4, 1). Find the work done.

CALCULUS

Note: Attempt 3 questions from this section.

3.

- i. Find the derivative by the first principles at any point x in the domain D(f) of the function $f(x) = \cos^2 x$.
- ii. Evaluate any Two of the following:

$$(a) \lim_{x \rightarrow \infty} \frac{x^2 - 5x + 2}{5x^2 + 6x - 4} \quad (b) \lim_{x \rightarrow 0} \frac{3e^x - e^{-x} - 2}{x}$$

$$(c) \lim_{\theta \rightarrow 0} \frac{\operatorname{cosec} \theta - \cot \theta}{\theta}$$

iii. Find $\frac{dy}{dx}$ of any Two of the following:

a) $y = (1nx)^{\tan^{-1}x}$

b) $x^y \cdot y^x = 1$

c) $x = \sin t^3 + \cos t^3, y = \sin t + 2\cos^{-1} t$

iv. Evaluate any two of the following:

a) $\int \sin 4y \sin 2y \, dy$

b) $\int_0^a \frac{dx}{(x^2 + a^2)^{3/2}}$

c) $\int \frac{dy}{\sqrt{4y - y^2}}$

v. By using differentials, calculate an approximate value of $\cos 44^\circ$.

OR Two polynomial functions f and g are defined by $f(x) = x^2 - 3x + 4$ and $g(x) = x + 1, \forall x \in \mathbb{R}$
Find $f \circ g$; $g \circ f$ and show that $f \circ g \neq g \circ f$.

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

Note attempt any two question:

4.

a) The vertices A, B and C of a triangle are (2, 1), (5, 2) & (3, 4) respectively. Find the coordinates of the circum-center and radius of the circumcircle of the triangle ABC.

b) Find the condition that conic $ax^2 + by^2 = 1$ should cut $a/x^2 + b/y^2 = 1$ orthogonally.

5.

(a) Evaluate any Two of the following:

(I) $\int \frac{dx}{x^2 + 4x + 5}$

(II) $\int e^x \frac{1 + \sin x}{1 + \cos x} \, dx$

(III) $\int \frac{\cos x \, dx}{\sin x (2 + \sin x)}$

OR $\int_0^\pi \tan^3 x \sec x \, dx$

(b) Prove that the angle between the conics $x^2 = 4ay$ and $y^2 = 4bx$ at a point other than the origin is:

$$\theta = \tan^{-1} \frac{3}{2} \left[\frac{a^{\frac{1}{3}} b^{\frac{1}{3}}}{a^{\frac{1}{3}} b^{\frac{1}{3}}} \right]$$

6.

(a) (i) Solve the following differential equation: $2 + 2y \frac{dy}{dx} = 1 + 3x^2, y(2) = 1$

(ii) Find the area above x-axis under the circle $x^2 + y^2 = 4$ and the ordinates $x = 1/2$ and $x = 3/2$.

(b) Find the relative maximum and minimum values of the function $f(x) = 2e^x + e^{-x}$.

Time: 20 Minutes

Max. Marks: 20

SECTION "A"(MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

(i) If $f: [-1, 5] \rightarrow \mathbb{R}$ is defined by $f(x) = x^2$ then $f(-2) =$:

- a) 4
- b) -2
- c) Undefined
- d) -4

(ii) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} =$:

- a) $e^2 \ln a$
- b) 1
- c) $\ln x$
- d) $\frac{1}{\ln x}$

(iii) The slope of a straight line which bisect the first and third quadrants is:

- a) 1
- b) 0
- c) -1
- d) ∞

(iv) The area of triangle whose vertices are (0,0) , (2,0) and (4, 0) is:

- a) 8 sq. units
- b) 4 sq. units
- c) 2 sq. units
- d) 1 sq. units

(v) If the equation of a straight line is $3x - y + 5 = 0$, then the point (1, 2) lies:

- a) above the line
- b) below the line
- c) on the line
- d) on both sides of the line

(vi) The radius of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is:

- a) $\sqrt{g^2 + f^2 + c}$
- b) $\sqrt{c - g^2 - f^2}$
- c) $\sqrt{g^2 - f^2 - c}$
- d) $\sqrt{g + f - c}$

(vii) The length of latus rectum of the parabola $x^2 = -28y$ is:

- a) 7
- b) 28
- c) 192
- d) -7

(viii) The equations of the directories of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are:

- a) $x = \pm a$
- b) $y = \pm a$

- c) $x = \pm a/e$
 d) $y = \pm 1/a$
- (ix) The vertex of the parabola $(x + 2)^2 = 4(y - 2)$ is:
 a) (-2, -2)
 b) (3, -2)
 c) (-2, 3)
 d) (-2, 2)
- (x) The point of concurrency of the medians of a triangle is called:
 a) In Centre
 b) Centroid
 c) Orthocenter
 d) Circumcenter

MATHEMATICS

2011

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) A is two-third the way from (1, 10) to (-8, 4) and B is the midpoint of (0, -7), (6, -11). Find the distance |AB|. Find the equation of the straight line which passes through the point (3, 4) and makes intercept on the axes such that the y-intercept is twice x-intercept.
- (ii) The point (2, 3) is the foot of the perpendicular dropped from the origin to a straight line. Write the equation of this line.
- (iii) Find the distance between the parallel lines $3x + 4y + 10 = 0$, $+8y - 9 = 0$.
- (iv) Find the equation of a circle with center at the point (1, -1) and touching the straight line $5x + 12 = 7$.
- (v) Find the equation of the parabola with focus (2, 3) and directrix $y - 5 = 0$.
- (vi) The length of the major axis of an ellipse is 25 units and its foci are the points $(\pm 5, 0)$. Find its equation.
- (vii) Find the eccentricity foci, directories and length of the latus rectum of the hyperbola $9x^2 - y^2 + 1 = 0$.
- (viii) A particle is acted upon by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$ and its displaced from the point $\hat{i} + 2\hat{j} - 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} - 3\hat{k}$ find the work done by forces.
- (ix) Prove that $[\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{c} + \vec{a}] = 2 [\vec{a} \vec{b} \vec{c}]$
- (x) Find the volume of the parallelepiped whose three adjacent edges are represented by the vectors:
 $\underline{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, $\underline{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\underline{c} = 3\hat{i} - \hat{j} + 2\hat{k}$.

CALCULUS

Note: Attempt 3 part questions from this section.

3.

- (i) Find the derivative by first principles at any point x in the domain D(f) of the function $f(x) = \cot x$.
- (ii) Evaluate any two of the following:

$$(a) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \quad (b) \lim_{x \rightarrow 1} \left(\frac{1}{1-x} - \frac{3}{1-x^3} \right) \quad (c) \lim_{x \rightarrow 0} \frac{e^{ax} - e^{bx}}{x}$$

(iii) Find $\frac{dy}{dx}$ of any two of the following :

d) $y = \tan^2 x + \sec x$

e) $2x^2 - 3xy + y^2 = 5$

f) $x = \ln t + \cos t, y = e^t + \sin t$

(iv) Using differentials, find the approximate value of $\cos 44^\circ$

(v) Show that $\sqrt{x + \Delta x}$ can be approximated to $\sqrt{x} + \frac{1}{2\sqrt{x}} \Delta x$. Hence find approximated value of $\sqrt{3.9}$.

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

Note attempt any two question:

4.

a) Show that the lines $x^2 - 4xy + y^2 = 0$ and $x + y = 3$ form an equilateral triangle. Also find the area of the triangle.

b) Find the equation of circle containing the point $(-1, -1)$ and $(3, 1)$ and with center on the line $x - y + 10 = 0$.

5.

a) Evaluate any Two of the following:

$$(i) \int \frac{2x \, dx}{(1+x^2)(3+x^2)} \quad (ii) \int \frac{x^3 \, dx}{\sqrt{a^2 - x^2}} \quad (iii) \int \frac{\sec x \tan x \, dx}{a + b \sec x}$$

b) Find the eccentricity, center, vertices and the foci of the ellipse given by the equation $4x^2 - 16x + 25y^2 + 200y + 316 = 0$.

MATHEMATICS

2010

Time: 20 Minutes

Max. Marks: 20

SECTION "A" (MULTIPLE CHOICE QUESTION)

1. Choose the correct answer for each from the given option:

(i) $\int e^{\tan x} \sec^2 x \, dx$ is:

- a) $e^{\sin x} + c$
- b) $e^{\sin 2x} + c$
- c) $e^{\tan x} + c$
- d) $\sec^2 x + c$

(ii) The least upper bound (l.u.b) of $\left\{-10, -5, 8, -\frac{1}{3}, 15, 21\right\}$ is:

- a) -10
- b) 8
- c) 15
- d) 21

(iii) The coordinate of centroid of the triangle whose vertices are $(2, 8)$, $(8, 2)$ and $(9, 2)$ are:

- a) $(3, 4)$
- b) $(19, 19)$

- c) $\left[\frac{19}{3} \quad \frac{19}{3}\right]$
d) $\left[\frac{1}{3} \quad \frac{1}{3}\right]$
- (iv) The inclination of x- axis is:
a) 90°
b) 45°
c) 0°
d) 270°
- (v) The distance of point (3, 2) from x-axis is:
a) $\sqrt{3}$ units
b) 5 units
c) 3 units
d) 2 units
- (vi) If two or more straight line meet at one point, then the lines are said to be:
a) Concurrent
b) Parallel
c) Perpendicular
d) Coincident
- (vii) Some of the slopes of the pair of line $ax^2 + 2hxy + by^2 = 0$ is:
a) $\frac{a}{b}$
b) $\frac{h}{b}$
c) $\frac{-h}{2a}$
d) $\frac{-2h}{b}$
- (viii) $\frac{d}{dx} (\operatorname{cosec}^{-1} y)$:
a) $\frac{-1}{y\sqrt{y^2-1}}$
b) $\frac{1}{y\sqrt{1-y^2}}$
c) $\frac{-1}{y\sqrt{1-y^2}}$
d) $\frac{1}{y\sqrt{y^2-1}}$
- (ix) An antiderivative of a function is also called:
a) Definite integral
b) Indefinite integral
c) Summation
d) Differential
- (x) The equations of the directories of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are:
a) $x = \pm a$
b) $y = \pm a$
c) $x = \pm a/e$
d) $y = \pm 1/a$

Time: 2 Hours 40 Minutes

Marks: 80

SECTION 'B' (SHORT - ANSWER QUESTIONS)

NOTE: Answer any 10 questions from this section. At least three question from each section.

SECTION B (Analytic Geometry) (Straight Line & Vector Algebra)

2.

- (i) If the line through (2, 5) and (-3, -2) is perpendicular to the line through (4, -1) and (x, 3), find x.
- (ii) Find the equation of the line which passes through the point (-3, 4) and has the sum of its equal to 1.
- (iii) Find the value of k when the vertices of the triangle are (2, 6), (6, 3) and (4, k) and area is 17 square units.
- (iv) The gradient of one of the lines $ax^2 + 2hxy + by^2 = 0$ is five times that of the other, show that $5b^2 = 9ab$.
- (v) Find the equation of the circle whose diameter is the latus rectum of the parabola $y^2 = -36x$.
- (vi) Find the eccentricity, foci and equations of directories of $25x^2 + 9y^2 = 225$.

OR Find the eccentricity of the hyperbola whose latus rectum is four times that of the transverse axis.

- (vii) Find the equation of the circle touching each of the axes in 4th quadrant at a distance of 6 from the origin.
- (viii) Find the equation of the circle which is concentric with the circle $x^2 + y^2 - 8x + 12y - 12 = 0$ and passes through the point (5, 4).
- (ix) Prove that $[\bar{a}, 2\bar{b} - 3\bar{c}, -2\bar{a} + \bar{b} + \bar{c}] = 5 [\bar{a}, \bar{b}, \bar{c}]$

OR Find the scalars x, y and z such that $x(3\hat{i} - 4\hat{k}) + y(-\hat{i} + \hat{j} + 2\hat{k}) + z(\hat{i} - 4\hat{k}) = (5\hat{i} + 4\hat{j} - 10\hat{k})$.

CALCULAS

NOTE: Attempt 3 questions from this section.

3.

- (i) Find the derivative by the 1st principles at $x = a$ in the domain D(f) of $(x) = \operatorname{cosec} x$.
- (ii) Evaluate $\lim_{n \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$
- (iii) Find $\frac{dy}{dx}$ of any two of the following:
 - (a) $y = x^{\sin x + \cos x}$
 - (b) $e^x \ln y = \sin^{-1} y$
 - (c) $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ at $\theta = \pi/2$

OR If $y = f(x) = a \cos x + b \sin x$, $\forall x \in \mathcal{R}$, show that $\frac{d^2 y}{dx^2} + y = 0$

(iv) Evaluate any two of the following:

$$(a) \int x \ln x \, dx \quad (b) \int_1^2 (3x^2 + 2x) \sqrt{x^3 + x^2 + 7} \, dx$$

$$(c) \int \sin 3x \cos 5x \, dx \quad \text{OR} \quad \int \frac{2x-3}{x^2+2x+2} \, dx$$

(v) Using differential, find the approximate value of $\cos 44^\circ$.

SECTION 'C' (DETAILED - ANSWER QUESTIONS)

Note attempt any two question:

4.

(a) (i) D, E, F are the mid-points of the sides BC, CA, AB respectively of the triangle ABC show that $AABC = 4ADEF$.

(ii) Find the equation of the locus of a moving point such that the slope of the line Joining the point to A (1, 3) is three times that of the slope of the line Joining the point to B (3, 1)

(b) Prove that two circles $x^2 + y^2 + 2gx + c = 0$ and $x^2 + y^2 + 2fy + c = 0$ touch each other, if $\frac{1}{f^2}, \frac{1}{g^2}, \frac{1}{c}$

5.

(a) Evaluate any Two of the following:

$$(i) \int x^2 \sqrt{4+x} \, dx \quad (ii) \int \frac{\cos x \, dx}{\sin x (2 + \sin x)} \quad (iii) \int \frac{\tan x}{\ln \cos x} \, dx$$

(b) Show that the eccentricities e_1 and e_2 of two conjugate hyperbolas satisfy the relation $e_1^2 + e_2^2 = e_1^2 \cdot e_2^2$.

6.

(a) (i) Show the differential equation: $y \frac{dy}{dx} = x (y^4 + 2y^2 + 1)$, $y(-3) = 1$

(ii) Find the area above the x-axis between the ordinates $x = \pi/4$ and $x = \pi/3$ under the curve $y = \tan x$

(b) Show that the maximum value of $f(x) = -\ln x$ is 1.